1. (a)

(b) After 4\textsuperscript{th} iteration, \(x_{\text{root}} = 1.4987\) with relative approximate percent error 0.05%.

(c) After 4\textsuperscript{th} iteration, \(x_{\text{root}} = 1.4987\) with relative approximate percent error 0%.

2. 1\textsuperscript{st} alternative: \(g(x) = 1 + \frac{\sin x}{2}\)

2\textsuperscript{nd} alternative: \(g(x) = 3x - 2 - \sin x\)

1\textsuperscript{st} alternative: after 4\textsuperscript{th} iteration, \(x_{\text{root}} = 1.4987\) with relative approximate percent error 0.03%.

2\textsuperscript{nd} alternative: after 4\textsuperscript{th} iteration, \(x_{\text{root}} = -33.8\) with relative approximate percent error 69%.

In the 2\textsuperscript{nd} alternative, the \(|g'(x)| < 1\) condition cannot be satisfied, hence divergence occurs.

3. i.
ii. (a) When solved with a tolerance of $10^{-4}$, $x_{\text{root}} = 0.6932$ at 15th iteration.

(b) When solved with a tolerance of $10^{-4}$, $x_{\text{root}} = 0.6931$ at 15th iteration.

(c) When solved with a tolerance of $10^{-4}$, $x_{\text{root}} = 0.6931$ at 4th iteration.

iii. Newton-Raphson method is the fastest since it has quadratic convergence. Regula-Falsi is generally faster than bisection because the algorithm is improved.

4. (a) 1st alternative: \( g(x) = \frac{x}{2^3} + 3x^2 + x - 5 \)

2nd alternative: \( g(x) = \sqrt{\frac{5}{3} - \frac{1}{3} x^4} \)

3rd alternative: \( g(x) = \sqrt[4]{80 - 48x^2} \)

(b) 1st alternative is selected, after the third iteration $x_{\text{root}} = 30.410$, divergence occurred.

(c) By satisfying the $|g'(x)| < 1$ condition, the solution converges to a root.

5. Rate of convergence: \( \left| \frac{p_{n+1} - p}{p_n - p} \right|^k < \lambda \)

where $p$ is the real root, $p_n$ and $p_{n+1}$ are $n$th and the $n+1$th estimations.

$k = 1$ linear convergence

$k = 2$ quadratic convergence

6. In the turbulent flow of fluid in a smooth pipe, the frictional force $f$ can be calculated as follows:

\[
\sqrt{f} = 2 \log_{10} (\text{Re} \sqrt{f}) - 0.8
\]

Where $\text{Re}$ is the constant Reynolds number and $f$ is a positive value which is smaller than 0.1.

For $\text{Re} = 1000$, the frictional force is found by bisection method in the interval $[0.01, 0.99]$. $f$ is found as 0.035.

7. $\ln(3) \approx 1.0942$