STEEL MATERIAL MODELS

Steel is considered a homogeneous material with equal strength in tension and compression. Furthermore, it is a ductile material that can undergo significant plastic deformations. These physical characteristics make the description of the steel response less complicated than that of concrete. The selection of a steel material model depends on the desired accuracy, simplicity and efficiency, noting in particular the large number of integration points in a structural model with many global degrees of freedom.

In reinforced concrete beams and columns, the longitudinal steel is thin and long, so that the predominant effect is uniaxial; thus the use of a uniaxial steel material is sufficient. Even for a well known material such as steel, the tension test behavior can vary significantly depending on combination with other components. Below figure sketches the response of mild steel with high-strength steel used in critical structural components, and with tool steel. Mild steel is highly ductile and clearly exhibits an extensive yield region. Hi-strength steel is less ductile and does not show a well defined yield point. Tool steel has little ductility, and its behavior displays features associated with brittle materials. The trade off between ductility and strength is typical. Note, however, that all three grades of steel have approximately the same elastic modulus, which is the slope of the stress-strain line in the linear region of the tension test.

Multi-axial stress-strain relations for steel are necessary in 2d and 3d finite element models, and are also important for the consideration of coupling of normal and shear stresses in beam finite element models.
1. Uniaxial Steel Models

Many of the uniaxial material models for steel were developed for the simulation of the behavior of reinforcing steel. Several effects such as yielding, nonlinear hardening and even buckling of the reinforcement (which is a nonlinear geometric behavior) are considered through phenomenological stress-strain relations.

In modeling uniaxial nonlinear response of steel, the bilinear model is the simplest. This model is widely used for the analysis of steel under monotonic loading. Under cyclic loading conditions the Bauschinger effect needs to be included: upon loading past the yield point, and unloading and reloading in the opposite direction the onset of yielding takes place at a stress that is lower than the initial yield stress.

In the bilinear steel model the stress-strain relation before yielding is

\[ \sigma = E \varepsilon \]  

(1)

where \( \sigma \) is the uniaxial stress, \( \varepsilon \) the uniaxial strain and \( E \) the Young modulus. To express the steel stress past the yield point the strain hardening modulus \( E_h \) is required

\[ \sigma = \sigma_y + E_h (\varepsilon - \varepsilon_y) \]  

(2)

The following expressions define the yield strain \( \varepsilon_y \) and permit the estimation of an average strain hardening modulus from the ultimate strength and corresponding strain of the reinforcing steel

\[ \varepsilon_y = \frac{\sigma_y}{E}; \quad E_h = \frac{\sigma_u - \sigma_y}{\varepsilon_u - \varepsilon_y} \]  

(3)

1.1 Plasticity Theory for Uniaxial Response of Steel

The models developed within the plasticity theory are actually proposed for defining the stress-strain relations of materials under complex loading conditions in 2d/3d rather than simply for 1d case. The equations proposed for the 3d case can be simplified for the uniaxial loading conditions and a steel material model within this theory can be formulated.

An in-depth discussion of the plasticity theory was presented for 1d case in the class. An elementary model for the uniaxial behavior of steel within this theory may be expressed by the following equations:

1) Total strain is divided into elastic and plastic parts: \( \varepsilon = \varepsilon_e + \varepsilon_p \)

2) Stress is calculated from elastic strain: \( \sigma = E \varepsilon_e = E (\varepsilon - \varepsilon_p) \)

3) State of admissible stresses defined by yield function:

\[ F(\sigma, \sigma_y, \alpha) = |\sigma - \sigma_y| - \sigma_y (\alpha) \leq 0 \quad \text{with} \quad \sigma_y (\alpha) = \sigma_{y0} + H_i \alpha \]

where \( \sigma_{y0} \) is the initial yield stress, \( H_i \) is the isotropic hardening modulus.
4) The rate of plastic strain is expressed with the plastic multiplier $\lambda$ in the direction of the loading as follows:

$$\dot{\varepsilon}_p = \lambda \frac{\partial F}{\partial \sigma} = \lambda \text{sign}(\sigma - \sigma_b)$$

5) Evolution of internal parameters ($\sigma_b$, the back-stress used for kinematic hardening and $\alpha$ for isotropic hardening) is calculated as follows:

$$\dot{\sigma}_b = H_k \dot{\varepsilon}_p; \quad \text{where } H_k \text{ is the kinematic hardening modulus}$$

$$\alpha = \int_0^t \dot{\varepsilon}_p \, dt \Rightarrow \dot{\alpha} = |\dot{\varepsilon}_p|$$

6) Kuhn-Tucker conditions:

$$\lambda \geq 0; \quad F \leq 0; \quad \lambda F = 0$$

7) Consistency condition:

$$\lambda \dot{F} = 0 \quad \text{when } F = 0$$

The evolution of back-stress in Item 5 above is not nonlinear; thus this may not realistically model the cyclic response of steel (Bauschinger effect). Nonlinear kinematic hardening can be incorporated as suggested by Armstrong and Frederick (1965) in plasticity models. In such an approach, the back-stress is modified by the inclusion of $H_{nl}$ nonlinear hardening parameter as follows:

$$\sigma_b = H_k \dot{\varepsilon}_p - H_{nl} |\dot{\varepsilon}_p| \sigma_b = H_k \lambda \text{sign}(\sigma - \sigma_b) - H_{nl} \lambda \sigma_b$$

Bauschinger effect in steel can be also incorporated with the bounding surface plasticity model originally proposed by Dafalias and Popov (1976), and mathematically formulated as a plasticity theory by Dafalias (1986). This theory has found significant applications in soil mechanics, as well as rubber bearings used in seismic isolation of buildings. In a later effort, Dafalias (1991) also presented an application of the bounding surface plasticity model for the cyclic response of steel.

In another effort, Lubliner et al. (1993) proposed a simple model of generalized plasticity theory for simulating the nonlinear kinematic hardening in metals. This model is briefly discussed later in this document as part of the 3d models.

### 1.2 GMP Steel Material Model:

One of the most popularly used uniaxial material model for steel is the hysteretic model proposed by Giuffre and Pinto and later implemented by Menegotto and Pinto (1973). The model is an empirical model, and is abbreviated as GMP model in literature. The attractive feature of the
model from the computational standpoint is the single equation that describes both loading and unloading. Nonetheless, the model is flexible enough to match sufficiently well experimental results of the hysteretic behavior of reinforcing steel. The stress-strain of the model is

\[
\sigma^* = b \varepsilon^* + \left( \frac{1-b}{1+\left( \varepsilon^* \right)^R} \right) \cdot \varepsilon^*
\]

where

\[
\sigma^* = \frac{\sigma - \sigma_r}{\sigma_0 - \sigma_r}; \quad \varepsilon^* = \frac{\varepsilon - \varepsilon_r}{\varepsilon_0 - \varepsilon_r}; \quad R = R_0 - \frac{a_1 \xi}{a_2 + \xi}; \quad b = \frac{E_h}{E}
\]

The single relation in Equation (4) defines an asymptotic transition from a straight line with slope \(E_0\) to a line with slope \(E_h\). The stress \(\sigma\) and strain \(\varepsilon\) correspond to the point of last reversal. The stress \(\sigma_0\) and strain \(\varepsilon_0\) correspond to the intersection point of the lines with slope \(E_0\) and \(E_h\) following a strain reversal. The parameter \(R\) in Equation (5) represents the curvature of the transition curve between the two line asymptotes and thus accounts for the Bauschinger effect. \(R\) depends on the normalized strain difference \(\xi\) between the maximum strain in the loading direction and \(\varepsilon_0\). The hardening slope \(E_h\) can be estimated from experimental data according to Equation (3).

Parameters \(R_0\), \(a_1\) and \(a_2\) are obtained by calibration with experimental results with the following values being common

\[
R_0 = 20.0; \quad a_1 = 18.5; \quad a_2 = 0.15
\]

Filippou (1983) has extended the GMP model to include isotropic hardening by imposing a shift in stress \(\sigma_{st}\) through moving the yield asymptote. This shift depends on the following relation

\[
\sigma_{st} = a_3 \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_y} - a_4 \right) \sigma_y
\]

where \(\varepsilon_{\text{max}}\) is the absolute maximum strain at the instant of strain reversal, and \(a_3\) and \(a_4\) are experimentally determined parameters with the following values being typical for cold formed reinforcing steel

\[
a_3 = 0.01; \quad a_4 = 7
\]

Below figure presents a typical stress-strain response obtained from GMP material model (Figure from OpenSees Manual).
1.3 Reinforcing Steel Material Models including Buckling:

Buckling is important for reinforcing steel bars with open length to bar diameter ratios larger than 5 (Monti and Nuti (1992)). In a recent effort, Kunnath et al. (2009) suggested a uniaxial reinforcing steel material model that incorporates buckling effects as a material response (see Reinforcingsteel uniaxial material in OpenSees). This model bases on the GMP material model discussed above. Including the buckling behavior, which is by the way a nonlinear geometric effect, as part of the material behavior would be an simplified approach with limitations.

2. Multiaxial Steel Models

The plasticity theory provides the suitable mathematical basis for the formulation of a three dimensional cyclic stress-strain response for structural steel. The simplest three- dimensional steel model is the $J_2$ plasticity model with linear isotropic and kinematic hardening rules (equivalent of the uniaxial model presented above). This model can simulate quite well the monotonic behavior of steel, but is not sufficiently accurate for the simulation of cyclic material response without the introduction of nonlinear kinematic hardening, either through a hardening rule, or by means of bounding surface plasticity theory.

2.1 $J_2$ Plasticity Model

The $J_2$ or Von Mises model is a basic material stress-strain relation in plasticity theory. The simplest form of the model makes use of the von Mises yield surface $F$

$$F = \sqrt{J_2} - k$$

(9)
where \( J_2 \) is defined as the second invariant of the deviatoric stress tensor \( s \), and \( k \) is a stress type parameter to be specified later. Yielding is characterized by the condition \( F = 0 \), and admissible stress states satisfy the condition \( F \leq 0 \).

The strain tensor is decomposed into an elastic and plastic part in small deformation theory

\[
\varepsilon = \varepsilon^e + \varepsilon^p
\]

The same decomposition is used for the deviatoric strain \( e \)

\[
e = e^e + e^p
\]

noting that the plastic part of the volumetric strain is zero. The elastic behavior of the material is described by the relation

\[
s = 2G(e - e^p)
\]

where \( G \) is the shear modulus. The yield surface under consideration of linear isotropic and kinematic hardening is

\[
F = \|s - \beta\| - \frac{2}{3} Y(\alpha) \quad \text{where} \quad Y(\alpha) = \sigma_y + H_i \alpha
\]

where \( \| \cdot \| \) is the Euclidean norm, \( \sigma_y \) is the uniaxial tensile yield strength, \( H_i \) is the isotropic hardening modulus, \( \alpha \) is the isotropic hardening variable, and \( \beta \) is the back-stress variable representing the distance of the yield surface center from the origin of deviatoric stress space.

The rate of change of plastic deformations and internal variables is defined by an associative rule

\[
\dot{\varepsilon}^p = \lambda \frac{\partial F}{\partial s} = \lambda \mathbf{n}, \quad \dot{\beta} = \frac{2}{3} \lambda H_k \mathbf{n}, \quad \text{and} \quad \dot{\alpha} = \frac{2}{3} \lambda
\]

where \( \mathbf{n} \) is the normal to the yield surface, \( H_k \) is the kinematic hardening modulus, \( \lambda \) is the plastic consistency parameter, and \( \xi \) is the relative stress defined by

\[
\xi = s - \beta
\]

with the following relation holding for the normal to the yield surface

\[
\mathbf{n} = \frac{\xi}{\|\xi\|}
\]

The consistency parameter \( \lambda \) satisfies the following loading/unloading conditions

\[
\lambda \geq 0, \quad F \leq 0 \quad \text{and} \quad \lambda F = 0
\]

Furthermore, the parameter \( \lambda \) needs to satisfy the consistency condition

\[
\lambda \dot{F} = 0
\]

The integration of the continuum equations with the backward-Euler integration scheme results in the radial return mapping algorithm (Simo and Hughes (1998))
2.2 Generalized $J_2$ Plasticity Model

This is a model proposed by Lubliner et al. (1993). Auricchio and Taylor (1995) discuss the advantage of this material model over a model with nonlinear hardening.

In generalized plasticity theory two functions describe the inelastic behavior of the material: the limit function $\tilde{F}$ and the yield function $F$. The latter is the same function as that used in classical plasticity theory. The limit function $\tilde{F}$ distinguishes between admissible and inadmissible states, and provides a smooth asymptotic transition between elastic and inelastic states during the loading process. Such behavior represents the cyclic response of metals more realistically.

Limit Function $\tilde{F}$ and Yield Function $F$

The stress-strain relations, the flow rule and the evolution rules of the $J_2$ plasticity model in Equations (10) to (17) apply directly to the generalized $J_2$ plasticity model. The only change is the introduction of the limit function that allows for the smooth transition from an elastic to a plastic state through the relation

$$\tilde{F} = h(F)\left(\frac{\partial F}{\partial \sigma}; \dot{\sigma}\right) - \lambda$$

where

$$h(F) = \frac{F}{\delta (\varphi - F) + H \varphi}; \quad H = H_i + H_k$$

$\varphi$ is the distance between the current and the asymptotic radius of the yield function, and $\dot{\sigma}$ is the rate of approaching the asymptote.

The integration of the rate equations for the generalized $J_2$ plasticity model with the backward-Euler integration scheme results in the algorithmic form of the equations.
REFERENCES
Dafalias, Y. F. "Bounding surface plasticity model for steel under cyclic loading." USA - Japan Seminar; Cyclic buckling of steel structures and structural elements under dynamic loading conditions, Osaka, Japan.