Example

\[ A_{AB} = 300 \text{ mm}^2 \]
\[ A_{BC} = 320 \text{ mm}^2 \]

ABC made from same material, \( E = 200 \text{ GPa} \).

Calculate the vertical deflection of point B \( (\delta_B)^V \).

Also determine the vertical stiffness of the system.

Joint B:

\[ F_{AB} = 32 \text{ kN} \quad \rightarrow \quad \delta_{AB} = \frac{32 \times 2303}{200 \times 300} = 1.23 \text{ mm} \quad (\text{elongation}) \]
\[ F_{BC} = -27.7 \text{ kN} \quad \delta_{BC} = \frac{-27.7 \times 2000}{200 \times 320} = -0.866 \text{ mm} \quad (\text{contraction}) \]

\[ |BB_1| = 1.23 \text{ mm} \]
\[ |BB_2| = 0.866 \text{ mm} \]

\( AB_1 \) and \( CB_2 \) intersect here at point \( B_3 \).

For small deformations, we will assume that arcs can be approximated by straight lines as follows. Thus point B moves to \( B_4 \) after deformation.

\[ |BB_4| = \delta_B \quad (\delta_B)^V = \frac{|B_3B_4|}{|B_3B_3|} \]

\[ \delta_B \cos \theta_2 \]

\[ 1.23 = \delta_B \cos \theta_1 \]

\[ 0.866 = \delta_B \sin \theta_2 \]

\[ \cos \theta_1 = 1.42 \]

\[ \frac{\cos \theta_1}{\cos \theta_2} \]

\[ \theta_1 + \theta_2 + 30 = 180 \implies \theta_1 = 150 - \theta_2 \]

\[ \frac{\cos (150 - \theta_2)}{\cos \theta_2} = \frac{\cos 150 \cos \theta_2 + \sin 150 \sin \theta_2}{\cos \theta_2} = 1.42 \]

\[ \cos 150 + \sin 150 + \tan \theta_2 = 1.42 \]

\[ \tan \theta_2 = 4.57 = \frac{(\delta_B)^V}{0.866} \quad \Rightarrow \quad (\delta_B)^V = 3.96 \text{ mm} \quad (\text{e}) \]

Vertical stiffness

\[ k = \frac{P}{(\delta_B)^V} = \frac{16 \text{ kN}}{3.96 \times 10^{-3} \text{ m}} = 4040.4 \text{ kN/m} \quad (B_2B \text{ distance}) \]