(d) COMPRESSIBILITY AND CONSOLIDATION

**D1.** A normally consolidated clay has the following void ratio $e$ versus effective stress $\sigma'$ relationship obtained in an oedometer test.

(a) Plot the $e - \sigma'$ curve.

(b) Plot the $e - \log \sigma'$ relationship and calculate the compression index.

<table>
<thead>
<tr>
<th>Effective Stress $\sigma'$ (kN/m$^2$)</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Void ratio, $e$</td>
<td>0.97</td>
<td>0.91</td>
<td>0.85</td>
<td>0.81</td>
<td>0.75</td>
</tr>
</tbody>
</table>

**D2.** A 6-m deep layer of sand overlies a 4m thick clay layer. The clay layer is underlain by sandy gravel. The water table is at the ground surface and the saturated unit weight for both the sand and the clay is 19 kN/m$^3$. A 3-m thick layer of fill (unit weight 20 kN/m$^3$) is to be placed rapidly on the surface over an extensive area. Assume that the data given in Problem 1 corresponds to that of a representative sample from the clay, whose coefficient of consolidation is 2.4 m$^2$/year.

(a) Calculate the total and effective vertical stresses and the pore water pressure at the centre of the clay layer before the fill is placed, immediately after the fill is placed, and after the clay has consolidated under the vertical stress increment due to the fill.

(b) Without subdividing the clay layer, calculate the final consolidation settlement due to the placement of the fill using

(i) \[ \frac{\Delta H}{H_0} = \frac{\Delta e}{1 + e_0}; \]

(ii) coefficient of volume compressibility;

(iii) compression index.

(c) What is the degree of consolidation $U_z$ at the centre of the clay layer when the pore water pressure at that depth is equal to 125 kN/m$^2$? What is the effective stress at that depth at that time?

(d) How long will it take to reach 50% average degree of consolidation $U$?

(e) What is the settlement at the end of 20 months?

(f) What time is required for 40 mm settlement?
(e) SHEAR STRENGTH (For this section, take \( g = 9.81 \text{ m/s}^2 \))

**E1.** Samples of compacted clean dry sand were tested in a 63 mm dia. shear box, and the following results obtained.

<table>
<thead>
<tr>
<th>Normal load (kgf)</th>
<th>16</th>
<th>32</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak shear load (N)</td>
<td>133.4</td>
<td>287.4</td>
<td>417.7</td>
</tr>
<tr>
<td>Ultimate shear load (N)</td>
<td>85.7</td>
<td>190.1</td>
<td>268.1</td>
</tr>
</tbody>
</table>

Determine the angle of shearing resistance of the sand (a) in the dense, and (b) in the loose state.

**E2.** In a mixed series of unconsolidated - undrained and consolidated - undrained triaxial tests with pore pressure measurement on the unsaturated, stiff, fissured Ankara Clay (average degree of saturation = 97 %), the following results have been obtained at failure.

<table>
<thead>
<tr>
<th>Test no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pore pressure (kPa)</td>
<td>-17</td>
<td>75</td>
<td>-17</td>
<td>-14</td>
<td>-2</td>
<td>-5</td>
</tr>
<tr>
<td>Cell pressure (kPa)</td>
<td>53</td>
<td>220</td>
<td>81</td>
<td>178</td>
<td>158</td>
<td>201</td>
</tr>
<tr>
<td>Deviator stress ((\sigma_1 - \sigma_3)) (kPa)</td>
<td>234</td>
<td>210</td>
<td>374</td>
<td>378</td>
<td>450</td>
<td>462</td>
</tr>
</tbody>
</table>

Determine the shear strength parameters in terms of effective stress (a) by drawing the average tangent to the Mohr circles; (b) by calculation from the modified shear strength envelope. State which method is preferable for such variable test results, and why.

**E3.** (a) By considering the torque on the curved (cylindrical) surface, and integrating the torque on ring-shaped elements on the two circular ends (neglecting the presence of the vane rod) of the sheared cylinder of soil, derive the following equation for the torque \( T \) required to shear a soft, saturated clay of shear strength \( c_u \), using a vane with rectangular blades of height \( h \) and diameter of circumscribing circle \( d \).

\[
T = \pi c_u \left( \frac{d^2 h}{2} + \frac{d^3}{6} \right)
\]

(b) A vane 75 mm in diameter and 150 mm long was used to measure the undrained shear strength of a soft clay. A torque of 50 Nm was required to shear the soil. The vane was then rotated rapidly to remould the soil completely. The ultimate torque recorded was 19 Nm. Determine the undrained shear strength of the clay in the natural and remoulded states, and hence find the sensitivity of the clay.

(c) If a 36 mm dia. undistributed specimen of the same clay as in Part (b) were tested in an unconfined compression test, what would be the axial load at failure, if the initial height is 72 mm and the specimen fails at an axial strain of 18 %?

**E4.** If a cylindrical specimen of saturated clay of initial height \( h_0 \) and initial cross-sectional area \( A_0 \) is subjected to an axial load under undrained conditions (either in the unconfined compression or in the triaxial compression test), it will undergo an axial shortening \( \delta h \) and its average cross-sectional area will increase to \( A \), but its volume will remain unchanged. By equating the initial volume of the specimen to its intermediate volume, prove the relationship.
\[ A = A_o \left( \frac{1}{1 - \varepsilon_a} \right) \]

where \( \varepsilon_a \) = axial strain.

**E5.** In an unconfined compression test on a saturated clay, the maximum proving ring dial reading recorded was 240x10^{-3} mm, when the axial shortening of the specimen, having an initial height of 70 mm and an initial diameter of 36 mm, was 12 mm. If the calibration factor of the proving ring was 3.2 N/10^{-3} mm, calculate the unconfined compressive strength and the undrained shear strength of the clay.

**E6.** The total vertical stress at a point P in a nearly saturated clay is 400 kPa and the pore pressure at P is 50 kPa. The pore pressure coefficients A and B of the clay have been measured as 0.4 and 0.8 respectively. Assuming the principal stress directions to remain horizontal and vertical, calculate the available shear strength on a horizontal plane at P when the load due to a structure results in an increase in total vertical stress at P of 80 kPa and an increase in total horizontal stress at P of 60 kPa. The shear strength parameters of the clay in terms of effective stress are \( c' = 8 \) kPa; \( \phi' = 24^\circ \).

**(f) LATERAL EARTH PRESSURE**

**F1.** The depth of soil behind a retaining wall is 8 m, and the soil properties are given in the figure below. A surcharge of 20 kN/m² is applied on the horizontal ground surface. Using the Rankine theory, plot the active pressure distribution behind the wall, and determine the total active thrust per m length of the wall.

![Soil Properties Diagram](image)

**F2.** A 7-m deep trench to be dug in a uniform, silty sand is supported by steel-sheet piling driven on either side of the trench, and supported by struts as shown. Such a system is normally in equilibrium if the total compression in the struts balances the active earth thrust, but if the compression in the struts continues to be increased, the sides may fail in passive resistance. The water table lies 3 m below the ground surface. The bulk unit weight of the soil is 16 kN/m³ above and 18 kN/m³ below the water table; the effective angle of friction \( \phi' = 35^\circ \) and cohesion \( c' = 12 \) kPa.

Plot the passive pressure distribution, and calculate the resultant compressive force in the struts per m length of the trench, for the sides to fail in passive resistance.
F3. Determine the total active thrust on the retaining wall shown in the figure below according to the Coulomb theory for the given trial failure plane. The unit weight of the soil is 20 \(\text{kN/m}^3\); the appropriate shear strength parameters are \(c_u = 10 \text{kN/m}^2\) and \(\phi_u = 25^\circ\); the angle of friction between the soil and the wall is 20\(^\circ\), and the wall adhesion is the same as \(c_u\).

(G1. A landslide has occurred along a slip surface parallel to the ground surface which was inclined at 15\(^\circ\) to the horizontal. The slip surface is at a vertical depth of 4 m, and the length of the slip measured along the slope is 200 m. Water in the soil may be assumed to extend to the ground surface and to be flowing parallel to it. The bulk unit weight of the soil is 18.5 \(\text{kN/m}^3\).

(a) Working from first principles, calculate the value of \(\phi'\) if \(c'\) is assumed as zero.

(b) What would have been the factor of safety if the soil had, in addition, \(c' = 5 \text{kPa}\)?

G2. A 30-degree slope is to be cut in sand having an angle of friction of 33\(^\circ\) and a unit weight of 17.5 \(\text{kN/m}^3\). There is no water table in the sand, but at a depth of 16 m below the horizontal ground surface there is a soft clay layer with an undrained shear strength of 14 kPa. The toe of the slope will lie 12 m below the ground surface.

(a) Calculate the factor of safety of the slope against the possibility of a translational slide along the top of the clay.
(D1)

<table>
<thead>
<tr>
<th>Normally Consolidated Clay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective Stress (kPa)</td>
</tr>
<tr>
<td>Void Ratio</td>
</tr>
</tbody>
</table>

a)

\[ e \text{ vs } \sigma' \]

b)

\[ e \text{ vs } \log \sigma' \]

- By using two effective stress values, that are on the linear portion of the curve, for example 200 and 300 kPa:
  \[ C_c = \frac{\Delta e}{\Delta \log \sigma} = \frac{e_{c_1} - e_{c_0}}{\log \sigma_1 - \log \sigma_0} = \frac{0.81 - 0.75}{\log^{100}_{200}} = 0.341 \]
Increase in stress due to rapidly placed fill = \( \Delta \sigma = 3 \times 20 = 60 \text{ kPa} \)  
\( \Delta u = 60 \text{ kPa} \) in clay

a) \textit{At the center of the clay layer};

(i) \textit{Before the fill is placed}:

\[ \sigma = 6 \times 19 + 2 \times 19 = 152 \text{ kPa} \]
\[ u_0 = 8 \times 10 = 80 \text{ kPa} \]
\[ \sigma' = \sigma - u = 152 - 80 = 72 \text{ kPa} \]

(ii) \textit{Immediately after the fill is placed}:

\[ \sigma = 152 + 3 \times 20 = 212 \text{ kPa} \]
\[ u = u_0 + u_{ie} = 80 + 60 = 140 \text{ kPa} \]
\[ \sigma' = \sigma - u = 212 - 140 = 72 \text{ kPa} \text{ (unchanged)} \]
\[ u_o = \text{initial pore water pressure} \]
\[ u_{ie} = \text{initial excess pore water pressure} \]

• For the "immediately after" case, the change in total stress should be applied to previous pore pressure values of clay (before the fill is applied). For example, before the fill is applied, the hydrostatic pore pressure at the center of the clay was 80 kPa, we apply the total stress change of 60 kPa to this value.

(iii) \textit{Long after the fill is placed}:

\[ \sigma = 152 + 3 \times 20 = 212 \text{ kPa} \]
\[ u = u_0 = 80 \text{ kPa} \]
\[ \sigma' = \sigma - u = 212 - 80 = 132 \text{ kPa} \text{ or } \sigma' = 72 + 60 = 132 \text{ kPa} \]

b) \( \text{at the center of the clay layer}; \)

\[ \sigma_0' = 72 \text{ kPa} \quad \rightarrow \quad e_0 = 0.944 \]
\[ \sigma_1' = 132 \text{ kPa} \quad \rightarrow \quad e_1 = 0.872 \quad \text{By using interpolation or from } e \text{ vs } \log \sigma' \text{ graph} \]
\[ H_0 = 4 \text{ m} \]

\[
\Delta H = \Delta e = \frac{e_0 - e_1}{1 + e_0}
\]

\[ S_c = \Delta H = 4 \times \frac{0.944 - 0.872}{1 + 0.944} = 0.148 \text{ m} = 148 \text{ mm} \]

\( S_c = \) consolidation settlement

(ii) \( m_v \) for the pressure range of \( \sigma'_o = 72 \text{ kPa} \) and \( \sigma'_1 = 132 \text{ kPa} \);

\[
m_v = \frac{1}{1 + e_0} \frac{e_0 - e_1}{\Delta \sigma} = \frac{1}{1 + 0.944} \frac{0.944 - 0.872}{132 - 72} = 6.17 \times 10^{-4} \frac{m^2}{kN}
\]

\[ S_c = m_v \times H_0 \times \Delta \sigma' = (6.17 \times 10^{-4}) \times 4 \times (132 - 72) = 0.148 \text{ m} = 148 \text{ mm} \]

(iii)

\[ S_c = \frac{C_c x \log(\sigma'_1/\sigma'_o)}{1 + e_0} x H_0 = \frac{0.341 x \log(132/72)}{1 + 0.944} x 4 = 0.185 \text{ m} = 185 \text{ mm} \]

• The result is different because pressure range of \( \sigma'_o = 72 \text{ kPa} \) and \( \sigma'_1 = 132 \text{ kPa} \) is not on the straight line portion of \( e \) vs \( \log \sigma' \) graph for which compression index (Cc) is used.

c)

\[ u = u_0 + u_e = 125 \text{ kPa} \]

\[ u_0 = 80 \text{ kPa} \]

\[ u_e = 125 - 80 = 45 \text{ kPa} \]

\( u_0=\) initial pore water pressure

\( u_e=\) excess pore water pressure at any time during consolidation process

\( u_{le}=\) initial excess pore water pressure= \( \Delta \sigma = 60 \text{ kPa} \)

\[ U = \frac{\text{Dissipation (decrease) in excess pore pressure at time "t"}}{\text{Initial excess pore pressure}} \]

\[ U = \frac{u_{le} - u_e}{u_{le}} = \frac{60 - 45}{60} = 0.25 = 25\% \] (Degree of consolidation at the center of the clay layer at that time)

\[ \sigma' = \sigma - u = 212 - 125 = 87 \text{ kPa} \] (at the center of the clay layer)
d) For $U = 50\% \rightarrow T_v = 0.197$ (From Degree of Consolidation Handout)

$d=$ max length of drainage path. For 2-way drainage path, $d=4/2=2$ m

$C_v=2.4 \text{ m}^2/\text{year}$

$$T_v = \frac{C_v t}{d^2} \rightarrow t = \frac{T_v d^2}{C_v} = \frac{0.197 \times 2^2}{2.4} = 0.328 \text{ year} \approx 4 \text{ months}$$

e) $T_v = \frac{C_v t}{d^2} = \frac{2.4 + \left(\frac{20}{12}\right)}{2^2} = 1.0 \rightarrow U \approx 93\%$ (From Degree of Consolidation Handout)

$t = 20 \text{ months} = 20/12 \text{ year}$

For 100% consolidation, settlement $= S_c = 0.148 \text{ m}$

For 93% consolidation, settlement $= S = 0.93 \times S_c = 0.93 \times 0.148 = 0.138 \text{ m} = 138 \text{ mm}$

f) $U = \frac{\text{Average Compression at time } "t"}{\text{Total Final Compression}} = \frac{40}{148} = 0.27 = 27\%$

For $U = 27\% \rightarrow T_v \approx 0.06$ (From Degree of Consolidation Handout)

$$t = \frac{T_v d^2}{C_v} = \frac{0.06 \times 2^2}{2.4} = 0.1 \text{ year} = 1.2 \text{ months}$$
Compacted Clean Dry Sand

Area of Shear Box, \( A = \frac{\pi d^2}{4} = \frac{\pi \times 0.063^2}{4} = 3.117 \times 10^{-3} \text{ m}^2 \)

Normal Stress, \( \sigma = \frac{P}{A} = \frac{P \times 9.81}{3.117 \times 10^{-3}} = 3147 \times \frac{P}{m^2} = 3.147 \times \frac{P}{m^2} \)

Shear Stress, \( \tau = \frac{S}{A} = \frac{S}{3.117 \times 10^{-3}} = 320.8 \times \frac{S}{m^2} = 0.3208 \times \frac{S}{m^2} \)

<table>
<thead>
<tr>
<th>Normal Load, ( P ) (kgf)</th>
<th>Normal Stress, ( \sigma ) (kPa)</th>
<th>Peak Shear Load, ( S_p ) (N) (given)</th>
<th>Peak Shear Stress, ( \tau_p ) (kPa)</th>
<th>Ultimate Shear Load, ( S_{ult} ) (N) (given)</th>
<th>Ultimate Shear Stress, ( \tau_{ult} ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>50.4</td>
<td>133.4</td>
<td>42.8</td>
<td>85.7</td>
<td>27.5</td>
</tr>
<tr>
<td>32</td>
<td>100.7</td>
<td>287.4</td>
<td>92.2</td>
<td>190.1</td>
<td>61.0</td>
</tr>
<tr>
<td>48</td>
<td>151.1</td>
<td>417.7</td>
<td>134</td>
<td>268.1</td>
<td>86.0</td>
</tr>
</tbody>
</table>

\( y = 0.8926x \)

\( y = 0.578x \)

Since this is a clean sand, the \( c' \)=0. The shear strength envelope of the sand for the peak and ultimate states are drawn by using excel.

Equation of the peak state (dense) is \( y=0.8926x \).

Equation of the ultimate state (loose) is \( y=0.5780x \).
From those equations the angle of shearing resistance for two states are calculated below.

\[ \phi_{\text{dense}}' = \arctan (0.8926) = 41.8^\circ \]
\[ \phi_{\text{loose}}' = \arctan (0.5780) = 30.0^\circ \]

**Note:** If you would like to measure the friction angle \( \phi' \) directly from the Normal stress and Shear Stress plot, using a protractor, the horizontal and vertical axes should be drawn with the same scale (e.g. the distance between [0-20] kPa in x-axis should be the same distance in y-axis for [0-20] kPa.)

In dense sand, the resisting shear increases with shear displacement until it reaches a failure stress \( \tau_f \). This \( \tau_f \) is called the peak shear strength. After failure stress is attained, the resisting shear stress gradually decreases as shear displacement increases until it finally reaches a constant value called ultimate shear strength.

In loose sand, the resisting shear stress increases with shear displacement until a failure shear \( \tau_f \) is reached. After that, shear resistance remains approximately constant for any further increase in shear displacement.
(E2) Unconsolidated – Undrained (UU) and Consolidated Undrained (CU) Triaxial Tests

Stiff Fissured Ankara Clay Sr = 97%

Center of Mohr Circles
\[ \frac{\sigma_1' + \sigma_3'}{2} = \frac{(\sigma_1 - u) + (\sigma_3 - u)}{2} = \frac{\sigma_1 + \sigma_3}{2} - u \]

Radius of Mohr Circles
\[ \frac{\sigma_1' - \sigma_3'}{2} = \frac{(\sigma_1 - u) - (\sigma_3 - u)}{2} = \frac{\sigma_1 - \sigma_3}{2} \]

u = Pore Pressure
\( \sigma_3 = \) Cell Pressure
\( \sigma_d = \sigma_1 - \sigma_3 = \) Deviator Stress

<table>
<thead>
<tr>
<th>Test No</th>
<th>u (kPa) (given)</th>
<th>( \sigma_3 ) (kPa) (given)</th>
<th>( \sigma_1 - \sigma_3 ) (kPa)</th>
<th>( \sigma_1 ) (kPa)</th>
<th>( \sigma_3 ) (kPa)</th>
<th>( \frac{\sigma_1' + \sigma_3'}{2} ) (kPa)</th>
<th>( \frac{\sigma_1' - \sigma_3'}{2} ) (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-17</td>
<td>53</td>
<td>234</td>
<td>287</td>
<td>70</td>
<td>304</td>
<td>187</td>
</tr>
<tr>
<td>2</td>
<td>75</td>
<td>220</td>
<td>210</td>
<td>430</td>
<td>145</td>
<td>355</td>
<td>250</td>
</tr>
<tr>
<td>3</td>
<td>-17</td>
<td>81</td>
<td>374</td>
<td>455</td>
<td>98</td>
<td>472</td>
<td>285</td>
</tr>
<tr>
<td>4</td>
<td>-14</td>
<td>178</td>
<td>378</td>
<td>556</td>
<td>192</td>
<td>570</td>
<td>381</td>
</tr>
<tr>
<td>5</td>
<td>-2</td>
<td>158</td>
<td>450</td>
<td>608</td>
<td>160</td>
<td>610</td>
<td>385</td>
</tr>
<tr>
<td>6</td>
<td>-5</td>
<td>201</td>
<td>462</td>
<td>663</td>
<td>206</td>
<td>668</td>
<td>437</td>
</tr>
</tbody>
</table>

a) By drawing the average tangent to the Mohr circles;

Shear strength parameters in terms of effective stress are \( c' = 36kPa \) and \( \phi' = 28^\circ \)

Prepared by Course Assistant Okan Koçkaya, Fall 2012
b) From the modified shear strength envelope with regression analysis to the 6 stress points,

\[
y = 0.5027x + 14.373 \\
R^2 = 0.8067
\]

A general representation of stress condition is in the below figure.

For \( x = 0 \); \( y = 14.373 \) from the equation \( y = 0.5027x + 14.373 \) obtained by applying regression to the 6 stress points. With the help of the equation \( y = 0.5027x + 14.373 \), the modified shear strength parameters can be calculated.
For x=0, \( a' = 14.4 \text{kPa} \) and \( \phi' = \arctan (0.5027) = 26.7^\circ \)

\( c' \) and \( \phi' \) can be calculated by:

\[
\phi' = \sin^{-1}(\tan a') = \sin^{-1}(\tan 26.7) = 30.2^\circ
\]

\[
c' = \frac{a'}{\cos \phi'} = \frac{14.4}{\cos 30.2} = 16.7 \text{kPa}
\]

Method (b) is preferable because it enables the use of statistical curve fitting techniques like linear regression.

(E3) Vane Shear Test

a)

\[
M_s = \text{shear strength} \times \text{area} \times \text{moment arm} = c_u \times \pi d h \times d/2 = c_u \times \frac{\pi d^2 h}{2}
\]

Resisting moment at the top:

\[
M_T = \text{shear strength} \times \text{area} \times \text{moment arm} = c_u \times \int_0^R 2\pi r dr \times d/2 = c_u \times \frac{\pi d^3}{12}
\]

Resisting moment at the bottom is equal to \( M_T \)

\[
M_B = c_u \times \frac{\pi d^3}{12}
\]

\[
T = M_s + M_T + M_B = c_u \times \frac{\pi d^2 h}{2} + c_u \times \frac{\pi d^3}{12} + c_u \times \frac{\pi d^3}{12} = \pi c_u \left( \frac{d^2 h}{2} + \frac{d^3}{6} \right)
\]

Prepared by Course Assistant Okan Koçkaya, Fall 2012
b) \( d = 75 \text{ mm} = 0.075 \text{ m} \)
\( h = 150 \text{ mm} = 0.15 \text{ m} \)
\( T = 50 \text{ Nm} = 0.050 \text{ kNm} \) (natural state)
\( T_{\text{ult}} = 19 \text{ Nm} = 0.019 \text{ kNm} \) (remolded state)

Shear strength of the soil in the natural state:
\[
T = \pi c_u \left( \frac{d^2 h}{2} + \frac{d^3}{6} \right)
\]
\[
0.050 = \pi c_u \left( \frac{0.075^2 \times 0.15}{2} + \frac{0.075^3}{6} \right)
\]
\( c_u = 32.34 \text{ kPa} \) (natural state)

Shear strength of the soil in the remolded state:
\[
T = \pi c_u \left( \frac{d^2 h}{2} + \frac{d^3}{6} \right)
\]
\[
0.019 = \pi c_u \left( \frac{0.075^2 \times 0.15}{2} + \frac{0.075^3}{6} \right)
\]
\( c_u = 12.29 \text{ kPa} \) (remolded state)

Sensitivity = \frac{\text{Undrained strength in the natural state}}{\text{Undrained strength in the remolded state}} = \frac{32.34}{12.29} = 2.6

\[\begin{align*}
&\text{c) Undisturbed clay specimen in an unconfined compression test} \\
&d_0 = 36 \text{ mm} = 0.036 \text{ m} \quad \text{(initial diameter)}
\end{align*}\]
\( h_0 = 72 \text{ mm} = 0.072 \text{ m} \) (initial height)
\( \varepsilon_f = 18\% = 0.18 \) (axial load at failure)
\( A_0 = \frac{\pi \times d_0^2}{4} = 1.02 \times 10^{-3} \text{ m}^2 \) (initial area)
\( A_f = \frac{A_0}{1 - \varepsilon} = \frac{\left(\pi \times 0.036^2\right)/4}{1 - 0.18} = 1.24 \times 10^{-3} \text{ m}^2 \) (corrected area at failure)
q_u = 2*c_u = 2*32.34 = 64.68 kPa  (compressive strength in an unconfined compression test)

The value of c_u = 32.3 kPa is calculated in the part b.

q_u = \sigma_1 - \sigma_3 \rightarrow 64.68 = \sigma_1 - 0 \rightarrow q_u = \sigma_1 = 64.68 kPa

P = \sigma_f * A_f = 64.68 \times 1.24 \times 10^{-3} = 0.0802 kN = 80.2 N  \text{ (axial load at failure)}

**(E4)**

**Saturated Clay**

initial volume = intermediate volume  (since its volume remains unchanged)

A_0 * h_0 = A * h

A = A_0 * (h_o/h)

To obtain (l_o/l):

\[ \varepsilon_a = \text{axial strain} \]

\[ \varepsilon_a = \frac{\delta h}{h_o} = \frac{h_o-h}{h_o} = 1 - \frac{h}{h_o} \rightarrow \frac{h}{h_o} = 1 - \varepsilon_a \rightarrow h_o = \frac{1}{1 - \varepsilon_a} \]

Then;

A = A_0 * (h_o/h) = A_0 * \left( \frac{1}{1 - \varepsilon_a} \right)

**(E5)**

**Unconfined compression test on a saturated clay**

\[ d_o = 36 \text{ mm} = 0.036 \text{ m} \quad \text{(initial diameter)} \]

\[ h_o = 70 \text{ mm} = 0.070 \text{ m} \quad \text{(initial height)} \]

\[ \delta h = 12 \text{ mm} = 0.012 \text{ m} \quad \text{(axial shortening at failure)} \]

\[ C_p = 3.2 \text{ N/}10^3\text{mm} \quad \text{(calibration factor of the proving ring)} \]

\[ \text{Dial reading} = 240*10^{-3} \text{ mm} \quad \text{(maximum proving ring dial reading)} \]

\[ \varepsilon_l = \delta h/h_o = 12/70 = 0.17 = 17\% \quad \text{(axial load at failure)} \]
\[ A_0 = \frac{(\pi d^2)}{4} = \frac{(\pi \times 0.036^2)}{4} = 1.018 \times 10^{-3} \text{ m}^2 \]  
(initial area)

\[ P_{\text{axial}} = \text{Dial reading} \times C_p = 240 \times 10^{-3} \times 3.2 \text{ N/10}^{-3} = 768 \text{ N} \]  
(axial force at failure)

\[ A = A_0 \times \left( \frac{1}{1 - e_n} \right) = 1.018 \times 10^{-3} \times \left( \frac{1}{1 - 0.17} \right) = 1.227 \times 10^{-3} \text{ m}^2 \]  
(corrected area at failure)

\[ \sigma_f = \frac{P_{\text{axial}}}{A} = \frac{768}{1.227 \times 10^{-3}} = 625917 \text{ Pa} = 625.9 \text{ kPa} \]

Unconfined compressive strength = \( \sigma_1 = q_u = 625.9 \text{ kPa} \)

Undrained shear strength = \( \sigma_1 / 2 = c_u = 313.0 \text{ kPa} \)

---

(E6) **Nearly saturated clay**

**@ point P:**

\[ \sigma_v = 400 \text{ kPa} \quad u = 50 \text{ kPa} \]
\[ \Delta \sigma_1 = 80 \text{ kPa} \quad \Delta \sigma_3 = 60 \text{ kPa} \]
\[ c' = 8 \text{ kPa} \quad \phi' = 24^\circ \]
\[ A = 0.4 \quad B = 0.8 \]

Pore pressure coefficients are used to express the response of pore water pressure to changes in total stress under undrained conditions.

In fully saturated soils, \( B \to 1 \) and in partially saturated soils, \( B < 1 \). In this problem, \( B = 0.8 \).

\[ \Delta u_1 = A \times B \times \Delta \sigma_1 \quad \Delta u_3 = B \times \Delta \sigma_3 \]
\[ \Delta u = \Delta u_1 + \Delta u_3 \]
\[ = B[\Delta \sigma_3 + A \times (\Delta \sigma_1 - \Delta \sigma_3)] = 0.8[60 + 0.4 \times (80 - 60)] = 54.4 \text{ kPa} \]

\[ u_{\text{final}} = u + \Delta u = 50 + 54.4 = 104.4 \text{ kPa} \]

\[ \sigma_{v(\text{final})} = \sigma_v + \Delta \sigma_1 = 400 + 80 = 480 \text{ kPa} \]

**@ point P:**

\[ \sigma'_{v(\text{final})} = \sigma_{v(\text{final})} - u_{\text{final}} = 480 - 104.4 = 375.6 \text{ kPa} \]

\[ \tau = c' + \sigma' \tan \phi' = 8 + 375.6 \times \tan 24^\circ = 175.2 \text{ kPa} \]
(F1)

\[ K_A = \frac{1 - \sin \phi}{1 + \sin \phi} \]

\[ K_{A1} = 0.36 \]

\[ K_{A2} = 0.24 \]

\[ P_A = K_A \gamma (\gamma + q) - 2 \gamma c x \sqrt{K_A} \]

<table>
<thead>
<tr>
<th>Soil</th>
<th>Depth (m)</th>
<th>Active Pressure (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>((0+20) \times 0.36 - 2 \times 10 \times 0.36)^{1/3} = -4.8)</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>((20+4 \times 19) \times 0.36 - 2 \times 10 \times 0.36)^{1/3} = 22.56)</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>((20+4 \times 19) \times 0.24 - 0 = 23.04)</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>((20+4 \times 19+4 \times (18-10)) \times 0.24 = 30.72)</td>
</tr>
</tbody>
</table>

Depth of Tension Crack: \( P_A = K_A \gamma (\gamma + q) - 2 \gamma c x \sqrt{K_{A1}} \)

For \( P_A = 0 \) \( z_0 = \frac{2 \gamma c x \sqrt{K_{A1}} - K_{A1} \gamma q}{\gamma \gamma K_A} = 0.7 \text{m} \)

“or \( z_0 \) can be calculated from the pressure diagram.”

Total active thrust:

\[ P_{\text{total}} = P_A + P \]

\[ P_{\text{total}} = 0.5x(22.56 \times 3.3) + 4 \times 23.04 + 0.5 \times 4 \times (30.72 - 23.04) + 0.5 \times 4 \times 40 = 224.74 \text{kN/m} \]

(into the page.)
Passive pressure is given by:

\[ P_p = K_p \gamma x \gamma z + 2 x c_x \gamma K_p \]

\[ = 3.69 \times [16 \times 3 + (18 - 10) \times 4] + 2 \times 12 \times \sqrt{3.69} = 341.3 \]

Note that, silty sand will be in drained condition below the water table, effective shear strength parameters and buoyant unit weights are used.

<table>
<thead>
<tr>
<th>Depth (m)</th>
<th>Passive Pressure (kN/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 + 2 \times 12 \times \sqrt{3.69} = 46.10</td>
</tr>
<tr>
<td>3</td>
<td>3.69 \times 16 \times 3 + 2 \times 12 \times \sqrt{3.69} = 223.22</td>
</tr>
<tr>
<td>7</td>
<td>3.69 \times [16 \times 3 + (18 - 10) \times 4] + 2 \times 12 \times \sqrt{3.69} = 341.3</td>
</tr>
</tbody>
</table>

Total Passive Resistance = Area(1) + Area(2) + Area(3)

\[ T \times \frac{46.10 + 223.22}{2} + 4 \times \frac{223.22 + 341.3}{2} + 4 \times \frac{40}{2} = 1613 \text{kN/m} \]
(F3)

\[ \gamma = 20 \text{kN/m}^3 \quad C_u = 10 \text{kN/m}^2 \quad \phi_u = 25^\circ \quad K_A = 0.406 \quad \delta = 20^\circ \]

Depth of tension cracks,

\[ z_0 = \frac{2C_u}{\gamma \sqrt{K_A}} \]

Assume tension zone extends as shown in the figure below:

\[ P_t = 50 \text{kN/m} \]

\[ C_{cd} = C_u \times CD = 10 \times 10.29 = 102.9 \text{kN/m} \]

\[ C_{cb} = C_u \times CB = 10 \times 6.43 = 64.3 \text{kN/m} \]

\[ W_{ABCDE} = A_{ABCDE} \times \gamma \]

\[ A_{ABCDE} = A1 + A2 + A3 + A4 = [6.3 \times 8/2] + [2 \times 2/2] + [2 \times 4.3] + [(2 + 1.57)/2 \times 0.3] = 36.34 \text{m}^2 \]

\[ W_{ABCDE} = 36.34 \times 20 = 726.84 \text{kN/m} \]
(a) Analytical Solution:

\[ \sum F_x = 0 \]

\[ P \cos 25^\circ + C_{cd} \cos 55^\circ - R \cos 60^\circ - C_{cb} \cos 85^\circ = 0 \]

\[ 0.5R - 0.906P = 102.9 \cos 55^\circ - 64.3 \cos 85^\circ = 53.42 \tag{1} \]

\[ \sum F_y = 0 \]

\[ W + P_1 - P \sin 25^\circ - C_{cd} \sin 55^\circ - R \sin 60^\circ - C_{cb} \sin 85^\circ = 0 \]

\[ 0.423P + 0.866R = 726.84 + 50 - 102.9 \sin 55^\circ - 64.3 \sin 85^\circ = 628.49 \tag{2} \]

Solving (1) and (2);

\[ P = 269 \text{ kN/m} \]

\[ R = 594.5 \text{ kN/m} \]

(b) Graphical Solution:
The total active thrust on the wall \((P_a)\) is equal to the vectorial sum of the \(P\) and \(C_{cb}\) forces in the opposite direction:

\[
P_a = P + C_{cb}
\]

So \(P_a\) is calculated as:

\[
P_a = 297 \text{ kN/m}
\]