Payback analysis (also called payout analysis) is another form of sensitivity analysis that uses a PW equivalence relation. Payback can take two forms: one for $i > 0\%$ (also called discounted payback) and another for $i = 0\%$ (also called no-return payback). The payback period $n_p$ is the time, usually in years, it will take for estimated revenues and other economic benefits to recover the initial investment $P$ and a specific rate of return $i\%$. The $n_p$ value is generally not an integer.

The payback period should be calculated using a required return that is greater than 0\%. In practice, however, the payback period is often determined with a no-return requirement ($i = 0\%$) to initially screen a project and determine whether it warrants further consideration.

To find the discounted payback period at a stated rate $i > 0\%$, calculate the years $n_p$ that make the following expression correct.

$$0 = -P + \sum_{t=1}^{t=n_p} NCF_t \left( \frac{P}{F}, i, t \right)$$

$NCF$ is the estimated net cash flow for each year $t$, where $NCF = receipts – disbursements$ (Net Cash-Flow). If the $NCF$ values are equal each year, the $P/A$ factor may be used to find $n_p$.

$$0 = -P + NCF \left( \frac{P}{A}, i, n_p \right)$$

After $n_p$ years, the cash flows will recover the investment and a return of $i\%$. If, in reality, the asset or alternative is used for more than $n_p$ years, a larger return may result; but if the useful life is less than $n_p$ years, there is not enough time to recover the initial investment and the $i\%$ return. It is very important to realize that in payback analysis, all net cash flows occurring after $n_p$ years are neglected. This is significantly different from the approach of all other evaluation methods ($PW$, $AW$ (annual worth), $ROR$, $B/C$) where all cash flows for the entire useful life are included. As a result, payback analysis can unfairly bias the selection of alternatives. Therefore, the payback period, $n_p$, should not be used as the primary measure of worth to select an alternative. It provides initial screening or supplemental information in conjunction with an analysis performed using the PW or AW method.

No-return payback analysis determines $n_p$ at $i = 0\%$. This $n_p$ value serves merely as an initial
indicator that a proposal is viable and worthy of a full economic evaluation. To determine the payback period, substitute $i = 0\%$ in the second equation and find $n_p$.

$$0 = -P + \sum_{t=1}^{t=n_p} NCF_t$$

For a uniform net cash flow series, the above equation is solved for $n_p$ directly.

$$n_p = \frac{P}{NCF}$$

An example use of no-return payback, as an initial screening of proposed projects, can be the case where a president of a corporation, who absolutely insists that every project must recover the investment in 3 years or less. Therefore, no proposed project with $n_p > 3$ at $i = 0\%$ should be considered further.

As with $n_p$ for $i > 0\%$, it is incorrect to use the no-return payback period to make final alternative selections. It neglects any required return, as the time value of money is omitted, and it neglects all net cash flows after time $n_p$, including positive cash flows that may contribute to the return on investment.

In general, the conclusions are: a 10% return requirement increases payback from 8 to 12 years; and when cash flows expected to occur after the payback period are considered, project return increases to 15% per year.

If two or more alternatives are evaluated using payback periods to indicate initially that one may be better than the others, the primary shortcoming of payback analysis (i.e., neglect of cash flows after $n_p$) may lead to an economically incorrect decision. When cash flows that occur after $n_p$ are neglected, it is possible to favor short-lived assets though longer-lived assets may produce a higher return. In these cases, PW or AW analysis should always be the primary alternative evaluation.

There are four important points to be understood about payback period calculations:

1. This is an approximate, rather than an exact, economic analysis calculation.
2. All costs and all profits, or savings of the investment, prior to payback are included without considering differences in their timing.
3. All economic consequences beyond the payback period are completely ignored.
4. Being an approximate calculation, payback period may or may not select the correct alternative. That is, the payback period calculations may select a different alternative from that found by exact economic analysis techniques.

But if payback period calculations are approximate, and are even capable of selecting the wrong alternative, why is the method used at all? The benefits of the payback period can be identified in twofold. Firstly, the calculations can be easily made by people unfamiliar with economic analysis, especially in analysis of no-return payback period. One does not need to know how to use gradient factors, or even to have a set of compound interest tables. Second, payback period is a readily understood concept.

Moreover, payback period does give us a useful measure, telling us how long it will take for the cost of the investment to be recovered from the benefits of the investment. Businesses and industrial firms are often very interested in this time period: a rapid return of invested capital means that it can be re-used sooner for the other purposes by the firm (Engineering Economic Analysis by Donald G. Newnan, 6th Edition, Engineering Press).

In summary, the payback period gives some measure of the rate at which a project will recover its initial outlay. This piece of information is not available from the present value or the internal rate of return. The payback period may not be used as a direct figure of merit, but it may be used as constraint: no project may be accepted unless its payback is shorter than some specified period of time.

**Example:** A company has two machine alternatives whose economic lives are 6 years. The price and annual income of these machines are given in the following table. According to the no return payback period, determine the alternative the company should invest.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Cost (TL)</th>
<th>Annual Income (TL/yr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Machine A</td>
<td>200.000</td>
<td>45.000</td>
</tr>
<tr>
<td>Machine B</td>
<td>300.000</td>
<td>60.000</td>
</tr>
</tbody>
</table>

Payback Period\(_1\) = 200.000/45.000 = 4.4 years
Payback Period\(_2\) = 300.000/60.000 = 5 years

According to the payback periods, first alternative should be preferred.
Example: A company wants to buy a production device for their new factory. They have two alternatives, whose cash flows are given in the following table. According to these cash flows, determine the no return payback period of these alternatives.

<table>
<thead>
<tr>
<th></th>
<th>Alternative A</th>
<th>Alternative B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>3.000.000 TL</td>
<td>3.500.000 TL</td>
</tr>
<tr>
<td>Annual Income</td>
<td>1.200.000 TL first year, decreasing by 300.000 TL per year thereafter</td>
<td>100.000 TL for the first year, increasing 300.000 TL per year thereafter.</td>
</tr>
<tr>
<td>Useful Life</td>
<td>4 years</td>
<td>8 years</td>
</tr>
</tbody>
</table>

**Alternative A**

<table>
<thead>
<tr>
<th>Years</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>-3.000.000</td>
<td>1.200.000</td>
<td>900.000</td>
<td>600.000</td>
<td>300.000</td>
</tr>
<tr>
<td>Cumulative Value</td>
<td>-3.000.000</td>
<td>-1.800.000</td>
<td>-900.000</td>
<td>-300.000</td>
<td>0</td>
</tr>
</tbody>
</table>

PB$_A$ = 4 years

**Alternative B**

<table>
<thead>
<tr>
<th>Years</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>-3.500.000</td>
<td>100.000</td>
<td>400.000</td>
<td>700.000</td>
<td>1.000.000</td>
<td>1.300.000</td>
</tr>
<tr>
<td>Cumulative Value</td>
<td>-3.500.000</td>
<td>-3.400.000</td>
<td>-3.000.000</td>
<td>-2.300.000</td>
<td>-1.300.000</td>
<td>0</td>
</tr>
</tbody>
</table>

PB$_B$ = 5 years

According to the payback periods, alternative A should be preferred.

Example: A construction company allocated a total of 18 million TL to a research on innovative construction methods that will improve their construction activities. The results are estimated to positively impact the net cash flow at the start of 6th year, and for the foreseeable future, at an average level of 6 million TL per year. As an initial screening for economic viability, determine both the no-return and i = 10% payback periods.
The NCF for years 1 through 5 is 0 TL and 6 TL million thereafter. Let \( x \) = the number of years beyond 5 when NCF > 0. For no-return payback. In TL million units,

\[ i = 0\%: \quad 0 = -18 + 5(0) + x \times 6 \]

\( n_p = 5 + x = 5 + 3 = 8 \) years

for \( i = 10\% \)

\[ i = 10\%: \quad 0 = -18 + 6 \times (P/A, 10\%, x) \times (P/F, 10\%, 5) \]

\[(P/A, 10\%, x) = \frac{18}{6 \times (0.6209)} = 4.8319 \]

\( x = 6.9 \) years

\( n_p = 5 + 7 = 12 \) years (rounded up)

**Example:** Compute the payback period of the following cash flow both for the no return and \( i = 10\% \) payback periods.

<table>
<thead>
<tr>
<th>End of year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>-100</td>
<td>25</td>
<td>30</td>
<td>25</td>
<td>-10</td>
<td>45</td>
<td>45</td>
</tr>
</tbody>
</table>

**a)** For \( i = 0\% \), 

\[ n_p = 4 + \frac{100- (25 + 30 + 25 - 10)}{45} = 4.66 \] years

**b)** 

\[ 0 = -100 + 25 \times (P/A, 10\%, 3) + 5 \times (P/F, 10\%, 2) - 10 \times (P/F, 10\%, 4) + 45 \times (P/A, 10\%, x) \]

\[ (P/A, 10\%, x) = \frac{4.5255}{30.735} = 1.3185 \quad \text{(by interpolation)} \]

\( n=1 \quad \Rightarrow \quad (P/A, 10\%, 1) = 0.9091 \)

\( n=2 \quad \Rightarrow \quad (P/A, 10\%, 2) = 1.7355 \)

\( x = 1 + \frac{1.3185 - 0.9091}{1.7355 - 0.9091} = 1.495 \)

\( n_p = 4 + 1.495 = 5.495 \) years

OR By trial and error

\( x = 1.483 \) years

\( n_p = 4 + 1.483 = 5.483 \) years

OR
Example: Compute the payback period of the following cash flow for $i = 10\%$ payback periods.

\[
\begin{array}{cccccc}
\text{End of year} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\text{Cash Flow} & -100 & 25 & 30 & 25 & -10 & 45 & 50 \\
\end{array}
\]

\[
\begin{array}{cccccc}
\text{Years} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
\text{Cash Flow} & -100 & 25 & 30 & 25 & -10 & 45 & 50 \\
\text{PW} & -100 & 22.73 & 24.79 & 18.78 & -6.83 & 27.94 & 28.225 \\
\text{Cumulative Present Worth} & -100 & -77.27 & -52.48 & -33.70 & -40.53 & -12.59 & 15.635 \\
\end{array}
\]

\[n_p = 5 + \frac{12.59}{12.59 + 15.635} = 5.446 \text{ years}\]

\textbf{NOTE:} You can use \(\text{P/A, i\%, x} \) expression if and only if you have an annual equivalent cash flow with \(x\) to be more than “1”. For other cases you should use percent contribution of present/future value of the last cash, as follow:

\textbf{Alternative Solution:}

\[
0 = -100 + 25 \left(\frac{P}{F}, 10\%, 1\right) + 30 \left(\frac{P}{F}, 10\%, 2\right) + 25 \left(\frac{P}{F}, 10\%, 3\right) - 10 \left(\frac{P}{F}, 10\%, 4\right) + 45 \left(\frac{P}{F}, 10\%, 5\right) + 50 \left(x\%\right) \left(\frac{P}{F}, 10\%, 6\right)
\]

\[
0 = -100 + 25 \left(0.9091\right) + 30 \left(0.8264\right) + 25 \left(0.7513\right) - 10 \left(0.6830\right) + 45 \left(0.6209\right) + 50 \left(x\%\right) \left(0.5645\right)
\]

\[
0 = -100 + 22.7275 + 24.7920 + 18.7825 - 6.8300 + 27.9405 + 28.2250 \left(x\%\right)
\]

\[
12,5875 = 28,2250 \left(x\%\right)
\]

\[x\% = 0.446\]

\[n_p = 5 + 0.446 = 5.446 \text{ years}\]
**Example:** A construction company wants to invest on a new regulator that produces electricity. The initial cost of the regulator is 10.5 million TL. The regulator will produce 40 million kwh with 81% efficiency per year. The government will buy the electricity at a price of 5 cent per kwh. After 30 years, the company will transfer the regulator to the government with no salvage value. If the MARR of the company is 12%, determine the payback period of this project.

\[ C = 10.5 \times 10^6 \text{ TL} \]

\[ A = 0.81 \times 40 \times 10^6 \times \frac{5}{100} = 1.62 \times 10^6 \text{ TL} \]

\[-10.5 \times 10^6 + 1.62 \times 10^6 \times (P/A, 12\%, PB) = 0\]

\[(P/A, 12\%, PB) = 10.5/1.62 = 6.4815 \text{ (trial and error by looking from interest tables)}\]

\[ PB \approx 13.05 \text{ years} \]

**Example:** A factory was bid at a cost of 25 million TL to a company. In addition, the company paid 5 million TL for the land of the factory and 16 million TL for the buildings associated with the factory. The useful life of the factory was determined as 30 years, and the salvage value of the factory was assumed to be 20 million TL. The average sales per year were determined as 2,500 units. The cost of the each unit is tabulated as follows:

<table>
<thead>
<tr>
<th>Cost</th>
<th>TL/unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>2,000</td>
</tr>
<tr>
<td>Workmanship</td>
<td>1,500</td>
</tr>
<tr>
<td>Energy</td>
<td>1,000</td>
</tr>
<tr>
<td>Management</td>
<td>200</td>
</tr>
<tr>
<td>Marketing</td>
<td>150</td>
</tr>
<tr>
<td>Finance</td>
<td>100</td>
</tr>
<tr>
<td>General Expenses</td>
<td>110</td>
</tr>
</tbody>
</table>

The price of the product : 10,000 TL/unit

By considering the MARR of the company as 25% per year, determine discounted payback period of the project.

\[ C = 25 \times 10^6 + 5 \times 10^6 + 16 \times 10^6 = (46 \times 10^6) \text{ TL} \]

\[ A = (10,000 - 2,000 - 1,500 - 1,000 - 200 - 150 - 100 - 110) \times 2,500 = 12,35 \times 10^6 \text{ TL/year} \]
46*10^6 = 12,35*10^6 (P/A, 25%, X)

(P/A, 25%, X) = 3,725 (trial and error by looking up interest tables)

X ≈ 12 years

**Example:** Consider a three-year project that has a two-year no-return payback period

a) Will the project NPV be positive?

b) Will the project IRR be higher than the cost of capital?

c) Should the project be accepted?

The answer to all parts of the question is: “we cannot know with the information provided.” Simply because, the project’s payback period tells us nothing about whether it generates a net positive return or that the return is higher than the cost of capital. Payback does not include the time value of money. Therefore, we do not know if we should accept the project or not.

**Example:** A company invested 1,600,000 TL. The cash flow of the investment is illustrated below. The no-return payback period for this investment is determined as 8 years. Determine the MARR, if the profit obtained at the end of the life of the investment is 297,440 TL.

\[
1,600,000 = X + (X+50,000) + (X+100,000) + (X+150,000) + \ldots + (X+350,000)
\]

\[
1,600,000 = 8X + 50,000 (1 + 2 + 3 + 4 + 5 + 6 + 7)
\]

\[
1,600,000 = 8X + 50,000 \times (28)
\]

X = 25,000 TL
297.440 = -1.600.000 (F/P, i%, 10) + [25.000 + 50.000 (A/G, i %, 10)] (F/A, i%, 10)

297.440 + 1.600.000 (F/P, i%, 10) - [25.000 + 50.000 (A/G, i %, 10)] (F/A, i%, 10) = 0

With trial and error:

i = 8%

297.440 + 1.600.000 * 2.159 - [25.000 + 50.000*3.8713]*14.487 = 585.488,85

i = 6%

297.440 + 1.600.000 * 1.791 - [25.000 + 50.000*4.0220]*13.181 = 182.815,90

i = 5%

297.440 + 1.600.000*1.629 - [25.000 + 50.000*4.099]*12.578 = 11.466,01

i = 4%

297.440 + 1.600.000*1.480 - [25.000 + 50.000*4.177]*12.006 = -142.163,1

MARR = 4 + [142.163,1 / (142.163,1 + 11.466,01) = 4,925%

**Example:** What must be the value of X and Y, according to the cash flow given below, if the discounted payback period of the cash flow is 7,6 years, and the profit at 9th year is 400.000 TL? The yearly interest rate is 10%.

\[
1.200.000 = X (P/A, 10\%, 5) + Y (P/A, 10\%, 1,6) (P/F, 10\%, 6)
\]

(P/A, 10\%, 1,6) = 1,4049 (by interpolation)

1.200.000 = 3,7908 X + Y (1,4049) (0,5645)
1.200.000 = 3,7908 X + 0,7931 Y (Equation 1)

$$400.000 = -1.200.000 \left( \frac{F}{P}, 10\%, 9 \right) + X \left( \frac{F}{A}, 10\%, 5 \right) \left( \frac{F}{P}, 10\%, 4 \right) + Y \left( \frac{F}{A}, 10\%, 3 \right)$$

$$400.000 = -1.200.000 \left( 2,3579 \right) + X \left( 6,1051 \right) \left( 1,4641 \right) + 3,3100 \ Y$$

3.229.480 = 8,9385 X + 3,3100 Y (Equation 2)

$$1.200.000 = 3,7908 X + 0,7931 Y$$

$$3.229.480 = 8,9385 X + 3,3100 Y$$

By solving these two equations simultaneously

$$-4,1735 \times (1.200.000 = 3,7908 X + 0,7931 Y)$$

$$-5.008.200 = -15,8209 X – 3,3100 \ Y \ 1^{st} \ equation$$

$$3.229.480 = 8,9385 X + 3,3100 \ Y \ 2^{nd} \ equation$$

$$6,8824 \ X = 1,778,720$$

$$X= 258,444,73 \ TL$$

$$1.200.000 = 3,7908 \left( 258,444,73 \right) + 0,7931 \ Y$$

$$Y= 277,755,29 \ TL$$

**Alternative Solution:**

$$400.000 = 0,40 \ Y \left( \frac{F}{P}, 10\%, 1 \right) + Y$$

$$400.000 = 0,40 \ Y \left( 1,1000 \right) + Y$$

$$400.000 = 1,44 \ Y$$

$$Y = 277,777,78 \ TL$$

$$1.200.000 = X \left( \frac{P}{A}, 10\%, 5 \right) + Y \left( \frac{P}{F}, 10\%, 7 \right) + 0,60 \ Y \left( \frac{P}{F}, 10\%, 8 \right)$$

$$1.200.000 = X \left( 3,7908 \right) + 277,777,78 \left( 0,5132 \right) + 277,777,78 \left( 0,60 \right) \left( 0,4665 \right)$$

$$1.200.000 = X \left( 3,7908 \right) + 220,305,56$$

$$979,694,44 = X \left( 3,7908 \right)$$

$$X = 258,440,02 \ TL$$
**Alternative Solution:**

<table>
<thead>
<tr>
<th>Years</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>- C₀</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>X</td>
<td>-</td>
<td>Y</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>PW</td>
<td>- C₀</td>
<td>0.9091X</td>
<td>0.8264X</td>
<td>0.7513X</td>
<td>0.6830X</td>
<td>0.6209X</td>
<td>0</td>
<td>0.5132Y</td>
<td>0.4665Y</td>
<td>0.4241Y</td>
</tr>
<tr>
<td>Cum. PW</td>
<td>- C₀</td>
<td>- C₀ + 0.9091X</td>
<td>- C₀ + 1.7355X</td>
<td>- C₀ + 2.4868X</td>
<td>- C₀ + 3.1698X</td>
<td>- C₀ + 3.7907X</td>
<td>- C₀ + 3.7907X</td>
<td>- C₀ + 0.5132Y</td>
<td>0.9797Y</td>
<td>1.4038Y</td>
</tr>
</tbody>
</table>

\( C₀ = 1,200,000 \) TL

**According to discounted payback period:**

Between years 7–8 \( (PW_{7,6} - PW_{7}) \): Change to PW=0) / (PW₈ - PW₇: Total Change in 1-year) = 0.60

\[
|0 + C₀ - 3,7907 X - 0,5132 Y| \times |0.4665 Y| = 0.60
\]

\[
(1,200,000 - 3,7907 X - 0,5132 Y) / 0,4665 Y = 0,60
\]

\[
1.200,000 - 3,7907 X - 0,5132 Y = 0,2799 Y
\]

\[
3,7907 X + 0,7931 Y = 1,200,000 \rightarrow 1^{st} \text{ equation}
\]

**According to the profit at the end of 9 years:**

\[
400,000 = (- C₀ + 3,7907 X + 1,4038 Y) \times (F/P, 10\%, 9)
\]

\[
400,000 = (- 1,200,000 + 3,7907 X + 1,4038 Y) \times (2,3579)
\]

\[
400,000 = - 2,829,480 + 8,9381 X + 3,3100 Y
\]

\[
8,9381 X + 3,3100 Y = 3,229,480 \rightarrow 2^{nd} \text{ equation}
\]

**Example**: Company A is considering three alternatives for investment, as shown below:

**Alternative 1**: \( P = -50,000 \) TL; \( n = 5 \) years; \( NCF = 28,000 \) TL per year

**Alternative 2**: \( P = -100,000 \) TL; \( n = 7 \) years; \( NCF = 70,000 \) TL for first and third year, decreasing by 10,000 TL per year thereafter

**Alternative 3**: \( P = -120,000 \) TL; \( n = 10 \) years; \( NCF = 40,000 \) TL for year 1, increasing by 2,500 TL per year thereafter
a) According to no-return payback period, which investment should be selected?

b) Calculate the internal rate of return for the cash flows of each project over its respective life. By considering the calculated internal rate of returns, which investment should be selected?

c) By considering the net present values (MARR of 12% per year), which investment should be selected?

d) According to the evaluations above, which alternative should the company invest? Why?

Alternative 1

Alternative 2

Alternative 3

\[ 1 + \frac{22,000}{28,000} = 1.786 \text{ years} \]

\[ 2 + \frac{30,000}{70,000} = 2.428 \text{ years} \]

\[ 2 + \frac{(120,000 - 82,500)}{45,000} = 2.833 \text{ years} \]

Select Alternative 1
b) For Alternative 1:

PW = -50,000 + 28,000 (P/A, i%, 5) = 0

With trial and error:

\( i = 40\% \)

\[ 50,000 + 28,000 \times 2,0351 = 6,984,59 \]

\( i = 45\% \)

\[ 50,000 + 28,000 \times 1,8755 = 2,514,77 \]

\( i = 50\% \)

\[ 50,000 + 28,000 \times 1,7366 = -1,374,49 \]

\[ 45 + \frac{2.514,77}{2.514,77 + 1.374,49} \times 5 = 48,23 \quad \text{ROR} = 48,23\% \]

For Alternative 2:

PW = -100,000 + (70,000 - 10,000 (A/G, i%, 5)) (P/A, i%, 5) (P/F, i%, 2) + 70,000 (P/F, i%, 1) = 0

With trial and error:

\( i = 40\% \)

\[ -100,000 + (70,000 - 10,000 \times 1,3580) \times 2,0351 \times 0,5102 + 70,000 \times 0,7143 = 8,583,77 \]

\( i = 45\% \)

\[ -100,000 + (70,000 - 10,000 \times 1,2980) \times 1,8755 \times 0,4756 + 70,000 \times 0,6896 = -859,32 \]

\[ 40 + \frac{8,583,77}{8,583,77 + 859,32} \times 5 = 44,54 \quad \text{ROR} = 44,54\% \]

For Alternative 3:

PW = -120,000 + (40,000 + 2.500 (A/G, i%, 10)) (P/A, i%, 10) = 0

With trial and error:

\( i = 40\% \)

\[ -120,000 + (40,000 + 2,500 \times 2,1419) \times 2,4136 = -10,533,1 \]

\( i = 35\% \)

\[ -120,000 + (40,000 + 2,500 \times 2,3337) \times 2,715 = 4,442,37 \]

\[ 35 + \frac{4,442,37}{10,533,1 + 4,442,37} = 36,48 \quad \text{ROR} = 36,48\% \]

Select Alternative 1
c) For Alternative 1:

\[-50,000 + 28,000 \times (P/A, i\%, 5)\]
\[-50,000 + 28,000 \times (P/A, 12\%, 5)\]
\[-50,000 + 28,000 \times 3,6048 = 50,933,73\]

\[A = 50,933,73 \times (A/P, 12\%, 5) = 50,933,73 \times 0,2774 = 14,290,017 \text{ TL/yr}\]

Common multiple of years: 70 years

\[\text{PW}(70) = 14,290,017 \times (P/A, 12\%, 70) = 14,290,017 \times 8,3303 = 117,698,95 \text{ TL}\]

For Alternative 2:

\[-100,000 + (70,000 - 10,000 \times (A/P, 12\%, 5)) \times (P/A, 12\%, 5) \times (P/F, 12\%, 2) + 70,000 \times (P/F, 12\%, 1)\]
\[-100,000 + (70,000 - 10,000 \times 1,7746) \times 3,6048 \times 0,8929 + 70,000 \times 0,8929 = 112,662,8\]

\[A = 112,662,8 \times (A/P, 12\%, 7) = 112,662,8 \times 0,2191 = 24,684,42 \text{ TL/yr}\]

\[\text{PW}(70) = 24,684,42 \times (P/A, 12\%, 70) = 24,684,42 \times 8,3303 = 205,628,62 \text{ TL}\]

For Alternative 3:

\[-120,000 + (40,000 + 2,500 \times (A/G, i\%, 10)) \times (P/A, i\%, 10)\]
\[-120,000 + (40,000 + 2,500 \times (A/G, 12\%, 10)) \times (P/A, 12\%, 10)\]
\[-120,000 + (40,000 + 2,500 \times 3,5846) \times 5,6502 = 156,644,1\]

\[A = 156,644,1 \times (A/P, 12\%, 10) = 156,644,1 \times 0,1770 = 27,726 \text{ TL/yr}\]

\[\text{PW}(70) = 27,726 \times (P/A, 12\%, 70) = 27,726 \times 8,3303 = 230,965,95 \text{ TL}\]

Select Alternative 3

d) Since the time value of money and cash flows that occur after PB are neglected in the no-return payback analysis, this method leads to a misleading alternative. Also, internal rate of return has a shortcoming in evaluation of mutually exclusive alternatives; therefore the projects should be evaluated by using incremental rate of return method. According to the other methods, the net present value method is a preferable method. Consequently, alternative 3 is determined as the best alternative among the alternatives.
**Example:** Which alternative should be selected according to

a) Discounted Payback Period Analysis?

b) Present Worth Comparison?

Take MARR = 10% per year (cash units are TL).

### Alternative 1

<table>
<thead>
<tr>
<th>End of year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>-150</td>
<td>50</td>
<td>50</td>
<td>-20</td>
<td>150</td>
</tr>
</tbody>
</table>

### Alternative 2

<table>
<thead>
<tr>
<th>End of year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>-100</td>
<td>50</td>
<td>40</td>
<td>20</td>
<td>30</td>
<td>-20</td>
<td>50</td>
</tr>
</tbody>
</table>

#### a) According to Discounted Payback Period:

<table>
<thead>
<tr>
<th>End of year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>-150</td>
<td>50</td>
<td>50</td>
<td>-20</td>
<td>150</td>
</tr>
<tr>
<td>(P/F,10%,n)</td>
<td>-</td>
<td>0,9091</td>
<td>0,8264</td>
<td>0,7513</td>
<td>0,6830</td>
</tr>
<tr>
<td>PW</td>
<td>-150</td>
<td>45,455</td>
<td>41,320</td>
<td>-15,026</td>
<td>102,45</td>
</tr>
<tr>
<td>Cum. PW</td>
<td>-150</td>
<td>-104,545</td>
<td>-63,225</td>
<td>-78,251</td>
<td>24,199</td>
</tr>
</tbody>
</table>

\[ n_p = 3 + \frac{78,251}{78,251 + 21,199} = 3,764 \text{ years} \]

#### b) According to Present Worth Comparison (NPV):

<table>
<thead>
<tr>
<th>End of year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Flow</td>
<td>-100</td>
<td>50</td>
<td>40</td>
<td>20</td>
<td>30</td>
<td>-20</td>
<td>50</td>
</tr>
<tr>
<td>(P/F,10%,n)</td>
<td>-</td>
<td>0,9091</td>
<td>0,8264</td>
<td>0,7513</td>
<td>0,6830</td>
<td>0,6209</td>
<td>0,5645</td>
</tr>
<tr>
<td>PW</td>
<td>-100</td>
<td>45,455</td>
<td>33,056</td>
<td>15,026</td>
<td>20,490</td>
<td>-12,418</td>
<td>28,225</td>
</tr>
<tr>
<td>Cum. PW</td>
<td>-100</td>
<td>-54,545</td>
<td>-21,489</td>
<td>-6,463</td>
<td>14,027</td>
<td>1,609</td>
<td>29,834</td>
</tr>
</tbody>
</table>

\[ n_p = 3 + \frac{6,463}{6,463 + 14,027} = 3,315 \text{ years} \]

Since \( n_p \) (Alt 2) < \( n_p \) (Alt 1) \( \rightarrow \) **Alt 2 should be selected!**

### From tables of the previous step:
NPV (Alt 1) = 24,199 TL \Rightarrow \text{ for 4 years}

NPV (Alt 2) = 29,834 TL \Rightarrow \text{ for 6 years}

NPV (Alt 1)_{12\text{ years}} = 24,199 \left[ 1 + (P/F, 10\%, 4) + (P/F, 10\%, 8) \right] = 52,016 \text{ TL}

NPV (Alt 2)_{12\text{ years}} = 29,834 \left[ 1 + (P/F, 10\%, 6) \right] = 46,675 \text{ TL}

Since NPV (Alt 1) > NPV (Alt 2) \Rightarrow \text{Alt 1 should be selected!}

\textbf{Example:} A family is planning to deposit 2,400 TL/year in a bank with a 25\% interest rate. How many years will they deposit this amount, in order to gain 17,000 TL/year continuously starting after the year of making last payment? (Hint P=A\left[\frac{(1+i)^n-1}{i(1+i)^n}\right])

\lim_{n \to \infty} \frac{(1+i)^n-1}{i(1+i)^n} = \frac{1}{i}

P= 17,000/i = 17,000/0.25 = 68,000

2,400 \times (F/A, 25\%, n) = 68,000

\left[\frac{(1+0.25)^n-1}{0.25}\right] = 28.33

(1 + 0.25)^n - 1 = 7,0825

1,25^n = 8,0825

n \cdot \log (1.25) = \log (8,0825)

n = 9.37 \text{ years}